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Non-Violation of Energy Conditions in the Future Accelerated Universe Due to Quantum Effects

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Here, an accelerated phantom model for the late universe is explored, which is free from future singularity. It is interesting to see that this model exhibits strong curvature for all time in future, unlike models with 'big-rip singularity' showing high curvature near singularity time only. So, quantum gravity effects grow dominant as time increases in late universe too. More importantly, it is demonstrated that quantum corrections to FRW equations lead to non-violation of 'cosmic energy conditions' of general relativity, which are violated for accelerating universe without these corrections.

KEY WORDS: cosmic energy conditions; phantom model without future singularity and quantum corrections.

1. INTRODUCTION

Observations (Benoit *et al.*, 2003; de Bernardis *et al.*, 2000; Halverson *et al*, 2002; Hanany *et al.*, 2000; Mason *et al.*, 2003; Miller *et al.*, 1999; Page *et al.*, 2003; Perlmutter *et al.*, 1999; Riess *et al.*, 1998, 2004; Spergel *et al.*, 2003), around turn of the last century, indicate that we are living in a spatially flat and accelerated universe. Cosmic acceleration is endowed with *dark energy* having $-1 \leq \mathrm{w} = p/\rho < -1/3$ with energy density $\rho > 0$ and pressure $p < 0$. With the equation of state parameter $-1 \leq \mathrm{w} < -1/3$, "strong energy condition" (SEC) is violated. Later on, Caldwell advocated for $\mathrm{w} < -1$, which violates "weak energy condition" (WEC) also and proposed *phantom model*. This model ends with singularity in finite future time (Alam *et al.*, 2003; Bertolami *et al.*, 2004; Caldwell, 1999, 2002; Caldwell *et al.*, 2003; Carroll *et al.*, 2003; Cline *et al.*, 2003; Daly, 2002; Daly and Guerra, 2002; Daly *et al.*, 2002; Frampton, 2003; Frampton and Takahashi, 2003; Faraoni *et al.*, 2003; Hannestad and Mortsell, 2002; Melchiorri *et al.*, 2003; Sami and Toporesky, 2003; Schuecker *et al.*, 2002; Singh *et al.*, 2003; Noriji and Odintsov, 2003a,b;

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Ziaeepour, 2000, 2003) with big-smash. A comprehensive review on dark energy is available in Copeland *et al.* (2006). In Barrow (2004), Barrow emphasized that, to get finite future-time singularity, violation of WEC is not necessary for accelerated universe. This model is generalized in Elizalde *et al.* (2004). Lake (2004) has shown that such cosmologies violate 'dominant energy condition' (DEC), but satisfy 'null energy condition' (NEC), SEC and WEC.

In phantom models, having "big-rip singularity," curvature becomes very strong near t_s. In curved space-time, quantum corrections contain higher order terms of curvature. So, quantum gravity effects become dominant for a small interval $|t_s - t| <$ one unit of time (with t being the cosmic time measured in units GeV^{-1}) (Elizalde *et al.*, 2004; Nojiri and Odintsov, 2004a,b; Nojiri *et al.*, 2005; Srivastava, 2004). It is similar to models of the early universe, where curvature invariants are very strong. The role of quantum gravity for entropy, in phantom universe, is discussed in Brevik *et al.* (2004). In Srivastava (2004), avoidance of big-rip problem is shown without using quantum corrections. In Nojiri and Odintsov (2003), using modified gravity, it is shown that big-rip problem does not arise in the model with curvature dark energy.

Here, it is shown that, for curvature invariants to grow sufficiently strong, future singularity is sufficient but not necessary. In what follows, a phantom model of the accalerated universe is explored, where quantum effects grow dominant. This model is free from four types of future singularities mentioned in Nojiri and Odintsov (2005). In this model, it is shown that quantum corrections in Friedmann equations lead to *non-violation* of 'cosmic energy conditions' even in accelerated universe. Here, natural units ($\hbar = c = 1$) are used with Gev as the fundamental unit.

Having compatibility with experimental evidences (Benoit *et al.*, 2003; de Bernardis *et al.*, 2000; Halverson *et al*, 2002; Hanany *et al.*, 2000; Mason *et al.*, 2003; Miller *et al.*, 1999; Page *et al.*, 2003; Perlmutter *et al.*, 1999; Riess *et al.*, 1998, 2004; Spergel *et al.*, 2003), topology of the observable universe is given by

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \tag{1}$$

Based on "cosmological principle," stress-energy tensor for content of the universe has perfect fluid form. So, energy conditions are given as (Lake, 2004)

$$\text{NEC} \Longleftrightarrow \rho + p \geq 0, \tag{2}$$

$$\text{WEC} \Longleftrightarrow \rho \geq 0, \quad \rho + p \geq 0, \tag{3}$$

$$\text{SEC} \Longleftrightarrow \rho \geq 0, \quad \rho + 3p \geq 0, \tag{4}$$

and

$$\text{DEC} \Longleftrightarrow \rho 0, \quad \rho \pm p 0. \tag{5}$$

The action for the phantom scalar ϕ is taken as

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right], \tag{6a}$$

where potential

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 \tag{6b}$$

with m being the mass.

So, in the homogeneous universe given by Eq. (1), energy density ρ and pressure p are obtained as

$$\rho = -\frac{1}{2}\dot{\phi}^2 + V_0 + \frac{1}{2}m^2\phi^2 \tag{7}$$

and

$$p = -\frac{1}{2}\dot{\phi}^2 - V_0 - \frac{1}{2}m^2\phi^2 \tag{8}$$

Moreover, action (6a) and Eq. (6b) yield field equation for ϕ as

$$-\ddot{\phi} - 3H\dot{\phi} + m^2\phi = 0, \tag{9}$$

where $H = \dot{a}/a$.

Friedmann equations are obtained as

$$\frac{3H^2}{8\pi G} = \rho = -\frac{1}{2}\dot{\phi}^2 + V_0 + \frac{1}{2}m^2\phi^2 \tag{10a}$$

using ρ given by Eq. (7) and

$$\frac{3}{4\pi G}\left(\frac{\ddot{a}}{a}\right) = 2\left(\dot{\phi}^2 + V_0 + \frac{1}{2}m^2\phi^2\right), \tag{10b}$$

where $G = M_P^{-2}$($M_P = 10^{19}$ GeV is the Planck mass). Conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{11}$$

With ρ and p from Eqs. (7), (8), Eqs. (11) yield Eq. (9). Equations (10a) and (11) give Eq. (10b), so Eqs. (10a) and (10b) are not independent like Eqs. (9) and (11). Hence, it is enough to solve only Eqs. (9) and (10a).

Equations (10a) and (9) integrate to a well-behaved scale factor

$$a(t) = a_0 \exp\left[\frac{m\phi_0\sqrt{12\pi G}}{3}(t - t_0) + \frac{m^2}{6}(t - t_0)^2\right] \tag{12a}$$

and

$$\phi = \phi_0 + \frac{m}{\sqrt{12\pi G}}(t - \bar{t}_0) \tag{12b}$$

with

$$V_0 = \frac{m^2}{24\pi G}, \tag{13a}$$

$$m\phi_0 = H_0\sqrt{\frac{3}{4\pi G}} \tag{13b}$$

and $t_0 = 13.7\,\text{Gyr} = 6.6 \times 10^{41}\,\text{GeV}^{-1}$ being the present age of the universe. Here $H_0 = 2.32 \times 10^{-42}\,h\,\text{GeV}$ is the current Hubble's rate.

Moreover, Eq. (12a) yields

$$\frac{\ddot{a}}{a} > 0 \tag{14}$$

showing speeded-up expansion. Equations (10a) and (10b) imply

$$\rho > 0 \tag{15a}$$

and

$$\rho + 3p < 0. \tag{15b}$$

Also, from Eqs. (7), (10a,b,c) and (12a,b), it is obtained that

$$\rho + p = -\frac{\dot{H}}{4\pi G} = -\frac{m^2}{12\pi G} < 0 \tag{15c}$$

and

$$\rho - p = \frac{\dot{H} + 3H^2}{4\pi G} > 0 \tag{15d}$$

Thus, through classical approach, we get violation of all energy conditions mentioned in Eqs. (2)–(5).

Now, effect of quantum corrections to Eqs. (10b), (15c) and (15d) is discussed. Equation (12b) yields

$$H(t) = \frac{m\sqrt{12\pi G}}{3}\left[\phi_0 + \frac{m}{\sqrt{12\pi G}}(t - t_0)\right]. \tag{15e}$$

This equation shows that curvature increases with time $t > t_0$ for expansion with no future singularity. Moreover, Eqs. (7) and (12a,b) show growth of phantom energy density also. So, as discussed above, quantum effects are expected to be dominant for $t > t_0$ due to increase in curvature. It is unlike models having "big-rip singularity," where quantum effects dominate only for a small time interval near singularity (Elizalde *et al.*, 2004; Nojiri and Odintsov, 2004a,b; Nojiri *et al.*, 2005; Srivastava, 2004).

Upto adiabatic order 4, one-loop quantum corection to the action (6) is given by Mann *et al.* (1988/89)

$$\Gamma = \int d^4x\sqrt{-g}\Bigg[\frac{3m^4}{4} - \frac{m^2}{6}R + ln\,(\mu^2/m^2)\Bigg(\frac{m^4}{2} - \frac{m^2}{2}R + \frac{1}{30}\Box R + \frac{1}{180}R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} - \frac{1}{180}R^{\mu\nu}R_{\mu\nu} + \frac{1}{72}R^2\Bigg]. \tag{16}$$

So, quantum corrections to stress tensor is obtained as

$$\begin{aligned} T^q_{\mu\nu} = &-\frac{m^2}{3}[1 + ln(\mu^2/m^2)]\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + 2ln(\mu^2/m^2)\Bigg[\frac{1}{30}\Bigg(\Box R_{\mu\nu} \\ &-\frac{1}{2}g_{\mu\nu}\Box R\Bigg) - \frac{1}{2}g_{\mu\nu}R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta} + 2R_{\mu\alpha\beta\gamma}R_\nu^{\alpha\beta\gamma} - 4\Box R_{\mu\nu} \\ &+ 2R_{;\mu\nu} - 4R_{\mu\alpha}R^\alpha_\nu - 4R^{\alpha\beta}R_{\alpha\mu\beta\nu}\Bigg) \\ &-\frac{1}{180}\left(2R^\alpha_{\mu;\alpha\nu} - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R + 2R^\alpha_\mu R_{\alpha\nu} - \frac{1}{2}g_{\mu\nu}R^{\alpha\beta}R_{\alpha\beta}\right) \\ &+ \frac{1}{72}\left(2R_{;\mu\nu} - 2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}\right)\Bigg] \end{aligned} \tag{17}$$

leading to

$$\begin{aligned} \rho^q = T_0^{0(q)} = m^2[1 + ln(\mu^2/m^2)]H^2 + 2ln(\mu^2/m^2)\Bigg[- \frac{13}{10}H\ddot{H} \\ -\frac{23}{40}\dot{H}^2 - \frac{13}{5}\dot{H}H^2 - \frac{6}{5}H^4\Bigg] \end{aligned} \tag{18a}$$

and

$$-p^q = T_1^{1(q)} = T_2^{2(q)} = T_3^{3(q)}$$
$$= \frac{m^2}{3}\left[1 + ln(\mu^2/m^2)\right](2\dot{H} + 3H^2) + 2ln(\mu^2/m^2)\left[-\frac{9}{40}\frac{d\ddot{H}}{dt}\right.$$
$$\left. + \frac{29}{120}H\ddot{H} + \frac{31}{40}H\dot{H} + \frac{5}{6}\dot{H}^2 + \frac{4}{5}\dot{H}H^2 + \frac{6}{5}H^4\right] \tag{18b}$$

Using $H(t)$, given by Eq. (16), Eqs. (18a,b) look like

$$\rho^q = m^2\left[1 - \frac{11}{15}ln(\mu^2/m^2) - \frac{12}{5m^2}ln(\mu^2/m^2)H^2\right]H^2 - \frac{23}{180}m^4 ln(\mu^2/m^2) \tag{19a}$$

and

$$-p^q = \frac{m^2}{3}\left[1 + ln(\mu^2/m^2)\right]\left(2\frac{m^2}{3} + 3H^2\right)$$
$$+ 2ln(\mu^2/m^2)\left[\frac{5}{54}m^4 + \frac{31}{120}m^2H + \frac{4}{15}m^2H^2 + \frac{6}{5}H^4\right] \tag{19b}$$

Equations (16)–(19b) show that H, given by Eq. (15c), dominates quantum corrections. So, a drastic change, in the behaviour of matter, is obtained with the same geometry (with $a(t)$ from Eq. (12b)) on replacing ρ by $\rho + \rho^q$ and p by $p + p^q$ in Eqs. (10b), (15c) and (15d). These corrections yield

$$\rho + 3p = -\frac{3}{8\pi G}\left(\frac{m^2}{3} + H^2\right) - \rho^q - 3p^q \approx \left[-\frac{3M_P^2}{8\pi} + \frac{364\pi m^2}{45M_P^2}ln(\mu^2/m^2)\right.$$
$$\left.\times\left\{\phi_0 + \frac{mM_P}{\sqrt{12\pi}}(t - t_0)\right\}^2\right]H^2, \tag{20a}$$

$$\rho + p = -\frac{m^2}{12\pi G} - \rho^q - p^q \approx \frac{24}{5}ln(\mu^2/m^2)H^4 > 0, \tag{20b}$$

$$\rho - p = \frac{m^2}{12\pi G} + \frac{3H^2}{4\pi G} - \rho^q + p^q \approx \left[\frac{3M_P^2}{4\pi} - m^2(1 + ln(\mu^2/m^2)\right]H^2 \tag{20c}$$

as terms containing H^4 dominate. Here $G = M_P^{-2}$ is used.

Equation (20a) shows that $\rho + 3p > 0$ if

$$\frac{3M_P^2}{8\pi} < \frac{91}{45}ln(\mu^2/m^2)\{3H_0 + m^2(t - t_0)\}^2 \tag{21}$$

using ϕ_0 from Eq. (13b).

Setting $\mu = m\sqrt{e}$ (e stands for exponential) in Eq. (21), it turns out that for $t > t_0 + \frac{0.243 M_P}{m^2}$, $\rho > 0$, $\rho \pm p > 0$ and $\rho + 3p > 0$. It shows that energy conditions, violated without accounting for quantum effects, are restored using quantum corrections when $t > t_0 + \frac{0.243 M_P}{m^2}$.

Thus, it is obtained that a phantom model, without 'future singularity' and exhibiting accelerated expansion, brings back quantum gravity era for a long span of time in future, after early universe and subsequent development of the universe upto the present epoch. It is unlike models having 'big-rip singularity,' where these effects are dominant for a short time. Interestingly, here, restoration of energy conditions SEC, WEC, DEC and NEC is possible on taking quantum effects into account, which are violated without taking these corrections for a speeded-up universe. It is due to negative ρ^q and negative p^q as quantum corrections, given by Eqs. (18a,b), as these terms dominate ρ and p. It shows that dominance of quantum gravity effects may lead to accelerated expansion even when $\rho > 0$, $\rho \pm p > 0$ and $\rho + 3p > 0$ and it is, in absence of these corrections, decelerated expansion is obtained when these conditions are obeyed. In the absence of these corrections, acceleration is obtained when SEC breaks. It is justified by early universe model, having dominance of quantum gravity effect and accelerated expansion. Moreover, in radiation and matter dominated models, $H \sim t^{-1}$, so curvature decreases with time giving quantum gravity ineffective and decelerated expansion for $\rho + 3p > 0$ as $\ddot{a} < 0$ from Eqs. (10b) and (18a,b).

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